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Dynamical conductivity of a two-layer structure with electron–acoustic phonon coupling

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Abstract. The frequency-dependent conductivity (resistivity) of two isolated parallel electron layers is calculated. We take into account both the direct electron–electron interaction and the coupling between electrons through phonon exchange. It is found that the interlayer momentum relaxation due to the direct Coulomb interaction and the phonon exchange interaction has the same dependence on the layer separation. The results for the conductivity rests on the Kubo formula for conductivity and the Green function formalism. The numerical results for the temperature dependence of σ_{12} in the DC limit is presented. Furthermore, we have calculated the frequency dependence of σ_{12} in the zero-temperature limit.

1. Introduction

Recently, there has been considerable interest, both theoretically and experimentally, on the coupling effects of two isolated parallel quasi-two-dimensional electron systems [1–10]. For the simplest such structures, the double quantum well (DQW), recent experiments [1] have suggested that the interwell interaction can dramatically alter the single-electron levels in samples with thin tunneling barriers. Even in a DQW with negligible tunneling, new fractional filling states in the extreme quantum limit have been investigated based on the inter-layer electron–electron interaction [2, 3]. Such direct electron–electron interaction is also believed to be the dominant mechanism responsible for the interlayer momentum relaxation. Experimentally measured frictional drag voltage in one layer when the other layer is driven by an applied voltage shows a roughly T^2 dependence, confirming the dominance of direct Coulomb interaction between the layers [10]. However, the deviation from T^2 behaviour is still noticeable. To explain these deviations, phonon exchange interaction between layers is proposed [10, 11] and the DC current has been calculated using the Boltzmann equation formalism [11].

In this paper, we present a first-principles calculation of frequency-dependent conductivity for two such parallel isolated two-dimensional electron systems. We take into account both the electron–electron interaction and electron–phonon coupling. We find that, contrary to the result of [11], the inter-layer momentum relaxation rate R due to electron–phonon coupling has a similar strong dependence on the separation between the two layers a. The rate due to the direct Coulomb interaction is proportional to T^2 at $\omega = 0$ because of the phase space requirement. The rate due to the phonon exchange interaction depends non-monotonically on T. It goes to zero much faster than T^2 at low temperatures and becomes nearly independent of T at high temperatures. At zero temperature, the conductivity goes to zero at low frequencies roughly as ω^2. We also attribute the deviation from ω^2 to electron phonon coupling.

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2. Evaluation of the conductivity

Let us consider electrons of density n_1 and mass m_1 on layer 1 and electrons of density n_2 and mass m_2 on layer 2. These two layers are separated by a distance a and the direction perpendicular to the layers is chosen as the z direction. We shall use a simplified model in which electrons can only move in the x–y plane. The wavefunction on each layer can be written as

$$\phi(\boldsymbol{p}, \boldsymbol{r}, z) = \mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}}\xi(z - z_{1,2}) \tag{1}$$

where $\boldsymbol{p}$, $\boldsymbol{r}$ are, respectively, the two-dimensional (2D) momentum and position vector along the plane, $z_1 = 0$ and $z_2 = a$; $\xi(z)$ is defined in such a way that it gives a δ-function-like distribution

$$|\xi(z)|^2 = \delta(z).$$

The Hamiltonian of our two-layer electron–phonon system is given as

$$H = H_0 + H_1 \tag{2}$$

where

$$H_0 = \sum_{\boldsymbol{p}} \sum_{l=1,2} E_{p,l} a^{\dagger}_{p,l} a_{p,l} + \sum_{\boldsymbol{Q}} \omega_{\boldsymbol{Q}} b^{\dagger}_{\boldsymbol{Q}} b_{\boldsymbol{Q}} \tag{3}$$

where $E_{p,l} = p^2/2m_l$ and $\boldsymbol{Q} = (\boldsymbol{q}, q_z)$ is a three-dimensional vector because our 2D electrons only interact with the bulk phonon of the compound; H_1 consists of both electron–electron and electron–phonon interactions, given by

$$H_1 = \frac{1}{2} \sum_{\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q}} \sum_{l,l'=1,2} V_q(ll') a^{\dagger}_{\boldsymbol{p}+\boldsymbol{q},l} a^{\dagger}_{\boldsymbol{p}'-\boldsymbol{q},l'} a_{\boldsymbol{p}',l'} a_{\boldsymbol{p},l} + \sum_{\boldsymbol{Q}} \sum_{l=1,2} C_l(\boldsymbol{Q}) a^{\dagger}_{\boldsymbol{p}-\boldsymbol{q},l} a_{\boldsymbol{p},l} (b^{\dagger}_{\boldsymbol{Q}} + b_{-\boldsymbol{Q}}). \tag{4}$$

Here $E_{p,l} = p^2/2m_l$ is the kinetic energy of an electron having momentum $\boldsymbol{p}$, $\omega_{\boldsymbol{Q}}$ is the wavenumber-dependent phonon frequency and $a^{\dagger}_{p,l}$, $a_{p,l}$ ($b^{\dagger}_{\boldsymbol{Q}}$, $b_{\boldsymbol{Q}}$) represent, respectively, the electron (phonon) creation and destruction operators with momentum $\boldsymbol{q}(\boldsymbol{Q})$. The coupling term $V_q(ll')$ is the Fourier transform of the Coulomb interaction for planar electrons. The coupling between planar electron and bulk phonon is treated in the jellium model as

$$C_l(\boldsymbol{Q}) = \mathrm{i}\left(\frac{2\pi e^2 \omega_{\boldsymbol{Q}}}{Q^2}\right)^{1/2} \int \mathrm{d}z \mathrm{e}^{\mathrm{i}q_z z} \xi_l(z) \xi_l^*(z). \tag{5}$$

For a δ-function distribution, (5) becomes simply

$$C_l(\boldsymbol{Q}) = \mathrm{i}\left(\frac{2\pi e^2 \omega_{\boldsymbol{Q}}}{Q^2}\right)^{1/2} \mathrm{e}^{\mathrm{i}q_z z_{1,2}} \tag{6}$$

where $\omega_{\boldsymbol{Q}}$ is the phonon frequency at wavenumber $\boldsymbol{Q}$.

To evaluate the conductivity, we start from the Kubo formula for conductivity, which is

$$\sigma_{\mu\nu,ll'}(\omega) = \int_0^{\infty} \mathrm{d}t \mathrm{e}^{\mathrm{i}\omega t} \int_0^{\beta} \mathrm{d}\lambda \langle j_{\mu l}(t - \mathrm{i}\lambda) j_{\nu l'}(0) \rangle \tag{7}$$

where ω is the frequency of the electromagnetic wave and we have set $\hbar$ equal to unity for notational convenience. Here

$$j_{\mu l}(0) = \frac{e}{m_l} \sum_{p} p_\mu a^\dagger_{p,l} a_{p,l}$$

and

$$j(t) = e^{iHt} j(0) e^{-iHt}$$

is the current operator in the Heisenberg representation and the average of an operator is defined by

$$\langle O \rangle = \mathrm{Tr}\left\{ \exp\left[\beta \left(\Omega + \sum_{l=1,2} \mu_l N_l - H \right) \right] O \right\} \tag{8}$$

where H is the total Hamiltonian of the system and Ω is the free energy defined through following relation:

$$e^{-\beta\Omega} = \mathrm{Tr}\left\{ \exp\left[\beta \left(\sum_{l=1,2} \mu_l N_l - H \right) \right] \right\}.$$

In the above equations, μ_l and N_l are the chemical potential and number operator of layer l (1 or 2), and β is the inverse temperature in energy units. Equation (7) can be rewritten as

$$\sigma_{\mu\nu,ll'}(\omega) = \sigma^0_l(\omega)\delta_{\mu\nu}\delta_{ll'} + \sigma^1_{\mu\nu,ll'}(\omega) \tag{9}$$

where

$$\sigma^0_l(\omega) = \frac{ie^2 n_l}{m_l \omega} \tag{10}$$

$$\sigma^1_{\mu\nu,ll'}(\omega) = \frac{1}{\omega} \int_0^\infty dt e^{i\omega t} \langle [j_{\mu l}(t), j_{\nu l'}(0)] \rangle. \tag{11}$$

The calculation of the current–current correlations and the conductivity in the bulk case [12] 2D systems [13, 14] and superlattice structures [15] are all well documented and we shall not present here detailed derivations. Our basic treatments are that electron–electron interactions are treated in the self-consistent field approximation and electron–phonon collisions in the Born approximation. Under these approximations, we need only to consider the class of diagrams of figure 1. Our expression includes the full dynamical screening of the electron–phonon systems. The wavy line in figure 1 is the effective interaction of an electron with another electron on the same or different layers; this consists of both the direct Coulomb interaction as well as the interaction through phonon exchange. Analytically it can be written as the following integral equation:

$$v_{ll'}(q, \alpha_m) = V_q(ll') + \sum_{q_z} C_l(q, q_z) C^*_{l'}(q, q_z) D_Q(\alpha_m) + \sum_{l''} V_q(ll'') \Pi_{l''}(q, \alpha_m) v_{l''l'}(q, \alpha_m)$$
$$+ \sum_{q_z, l''} C_l(q, q_z) C^*_{l''}(q, q_z) D_Q(\alpha_m) \Pi_{l''}(q, \alpha_m) v_{l''l'}(q, \alpha_m) \tag{12}$$

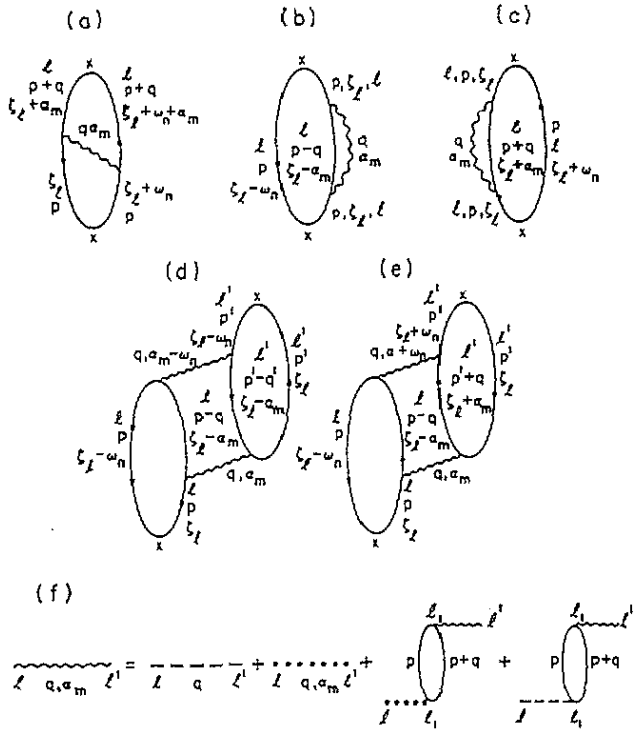

Figure 1. (*a*)–(*e*) A class of diagram that contributes to the high-frequency conductivity. The full curve represents the electron propagator and the wavy curve represents the effective interaction. (*f*) The diagrammatic representation of the effective interaction, where the broken line is for bare electron–electron interaction and the dotted line is for unscreened electron–phonon coupling.

where

$$\alpha_m = 2\pi m\mathrm{i}/\beta \qquad m = \text{any integer}.$$

In (11), $\Pi(q,\omega)$ is the electronic polarizability and D_Q is the free phonon propagator, given as

$$D_Q = \omega_Q/(\omega^2 - \omega_Q^2).$$

We now carry out the summation over q_z in (12) and define

$$\psi_{ll'}(q,\omega) = \sum_{q_z} C_l(q,q_z)C_{l'}^*(q,q_z)D_Q(\omega). \tag{13}$$

By making a suitable choice of the electronic wavefunction in the z direction, the summation over q_z can be performed in (13). In the present approximation electrons are purely two-dimensional, and we obtain

$$\psi_{ll'}(q,\omega) = \frac{2\pi^2 e^2}{q}\frac{\omega_q^2}{\omega^2-\omega_q^2}\mathrm{e}^{-q|l-l'|a}. \tag{14}$$

Since there are only two layers in our system, all components of $v_{ll'}$ can be obtained analytically; for example

$$v_{21}(q,\omega) = \frac{V_{21}+\psi_{21}}{[1-(V_{11}+\psi_{11})\Pi_1][1-(V_{22}+\psi_{22})\Pi_2]-(V_{12}+\psi_{12})(V_{21}+\psi_{21})\Pi_1\Pi_2} \tag{15}$$

where we have suppressed the explicit q dependence of V and the q, ω-dependence of ψ and Π. In the above equation, $V_{11} = 2\pi e^2/q$, $V_{21} = V_{11}e^{-qa}$.

The frequency-dependent interlayer interaction v_{12} can be separated into two terms

$$v_{21}(q,\omega) = \frac{V_{21}(q)}{\epsilon(q,\omega)} + \frac{\tilde{\psi}_{21}}{\epsilon(q,\omega)^2} \tag{16}$$

where $\epsilon(q,\omega)$ is the dynamical dielectric function of the electrons, given by

$$\epsilon(q,\omega) = (1 - V_{11}\Pi_1)(1 - V_{22}\Pi_2) - V_{12}V_{21}\Pi_1\Pi_2 \tag{17}$$

and the renormalized electron–phonon coupling $\tilde{\psi}$ is given as

$$\tilde{\psi}_{21}(q,\omega) = \frac{\psi_{21}(q,\omega)}{1 - P(q,\omega)/\epsilon(q,\omega)}\left[1 - \Pi_1\Pi_2\left(V_{11}V_{22} - V_{12}V_{21} - \psi_{11}V_{11} - V_{21}\psi_{21}\right)\right]. \tag{18}$$

The function $P(q,\omega)$ is the correction to the dielectric function due to electron–phonon coupling:

$$P(q,\omega) = \Pi_2(1 - V_{11}\Pi_1)\psi_{22} + \Pi_1(1 - V_{22}\Pi_2)\psi_{11} + \Pi_1\Pi_2\left[\psi_{11}\psi_{22} + \psi_{12}\psi_{21} + 2V_{12}\psi_{12}\right]. \tag{19}$$

The meaning of the separation in (16) is that the first term is the screened Coulomb potential due to the collective motion of the electrons and the second term represents the renormalized phonon interaction at a vertex with the electron through a screened electron–phonon interaction.

The renormalized electron–phonon coupling $\tilde{\psi}$, given by (18) and (19), looks rather complicated, especially when separation of the real part and imaginary part is required. However, the physical meaning of this renormalization can be visualized in the case of weak electron–electron coupling (or small plasma parameter $r_s = me^2/\hbar^2 k_F$). In this case, terms in higher order of r_s can be neglected (e.g. the $\Pi_1\Pi_2$ term) and we obtain

$$\tilde{\psi}_{21}(q,\omega) = \frac{2\pi e^2}{q}\frac{\omega_q^2}{\omega^2 - \omega_q^2(1+\Delta) - \mathrm{i}(\omega\delta + \omega_q^2\Gamma)}e^{-qa} \tag{20}$$

where δ is the halfwidth of the free phonon and

$$\Delta = \frac{2\pi e^2}{q}\mathrm{Re}\left(\frac{\Pi_1 + \Pi_2}{1 - V_{11}\Pi_1 - V_{22}\Pi_2}\right) \tag{21}$$

is the relative shift of phonon frequency, and

$$\Gamma = \frac{2\pi e^2}{q}\mathrm{Im}\left(\frac{\Pi_1 + \Pi_2}{1 - V_{11}\Pi_1 - V_{22}\Pi_2}\right) \tag{22}$$

is the correction to the phonon width due to electron–electron interaction.

In this paper, our main concern is the interlayer momentum or current relaxation due to both the electron–electron and electron–phonon interaction. Therefore we shall study the off-diagonal matrix element of the conductivity in detail:

$$\sigma_{\mu\nu,21}(\omega) = \frac{1}{\mathrm{i}\omega}Y^+_{\mu\nu,21}(\omega) \tag{23}$$

where for any function $f(z)$ in the complex z plane, we denote

$$f^{\pm}(\omega) = \lim_{z \to \omega \pm i0} f(z).$$

The function Y^+ is the analytical continuation of $M(n)$, which is given by

$$M_{\mu\nu,21}(n) = \int_0^{\beta} du e^{i2\pi nu/\beta} M_{\mu\nu,21}(u)$$

and

$$M_{\mu\nu,21}(u) = \langle T[j_{\mu 2}(u) j_{\nu 1}(0)] \rangle \qquad -\beta < u < \beta. \tag{24}$$

The interlayer matrix element $M_{\mu\nu,21}$ can be obtained using the Green function technique to evaluate diagrams 1(*c*) and 1(*d*). We shall also assume that system is isotropic on the layer ($M_{\mu\nu} = M\delta_{\mu\nu}$ and we denote $M_{\mu\mu,21}$ as simply M_{21}) to obtain

$$M_{21}(\omega_n) = \frac{1}{2\beta\omega_n^2} \int \frac{q^2 d^2 q}{4\pi} \frac{e^6}{m_2 m_1} \sum_m v_{21}(q, \alpha_m) v_{12}(q, \alpha_m + \omega_n) \times \left[\Pi_1(q, \alpha_m + \omega_n) - \Pi_1(q, \alpha_m)\right]\left[\Pi_2(q, \alpha_m + \omega_n) - \Pi_2(q, \alpha_m)\right] \tag{25}$$

where e^2 has been factored out from v_{21}. In order to perform analytical continuation on the upper-half z plane, we first sum over m. After some algebra, we obtain

$$M_{21}(\omega) = \frac{1}{2\omega_n^2} \int \frac{q^2 d^2 q}{4\pi} \frac{e^6}{m_2 m_1} \int du \coth(\tfrac{1}{2}\beta u) v_{12}(q, u^+ + \omega) \times \{v_{21}(q, u^+)[\Pi_1(q, u^+ + \omega) - \Pi_1(q, u^+)][\Pi_2(q, u^+ + \omega) - \Pi_2(q, u^+)] - v_{21}(q, u^-)[\Pi_1(q, u^+ + \omega) - \Pi_1(q, u^-)][\Pi_2(q, u^+ + \omega) - \Pi_2(q, u^-)]\} \tag{26}$$

where $u^{\pm} = u \pm i0$. In the following, we are only interested in absorption properties due to interlayer interactions. The interlayer relaxation time τ_{21} is given by the real part of the interlayer element of the conductivity tensor, $\tau_{21}^{-1} \sim \mathrm{Re}\left([\sigma]^{-1}\right)_{21}$. We obtain

$$\tau_{21}^{-1}(\omega) = \frac{\pi e^4}{2\omega(m_1 n_2 + m_2 n_1)} \int \frac{q^2 d^2 q}{4\pi} \int du \{\coth(\tfrac{1}{2}\beta u) - \coth[\tfrac{1}{2}\beta(u + \omega)]\} F(u, u + \omega) \tag{27}$$

where $F = F_1 + F_2 + F_3$, and each can be written explicitly as

$$F_1 = [\Pi_2^{\mathrm{I}}(u + \omega) + \Pi_2^{\mathrm{I}}(u)][\Pi_1^{\mathrm{I}}(u + \omega) + \Pi_1^{\mathrm{I}}(u)][v_{12}^{\mathrm{R}}(u + \omega) v_{21}^{\mathrm{R}}(u) + v_{12}^{\mathrm{I}}(u + \omega) v_{21}^{\mathrm{I}}(u)] - [\Pi_2^{\mathrm{I}}(u + \omega) + \Pi_2^{\mathrm{I}}(u)][\Pi_1^{\mathrm{R}}(u + \omega) - \Pi_1^{\mathrm{R}}(u)] \times [v_{12}^{\mathrm{I}}(u + \omega) v_{21}^{\mathrm{R}}(u) - v_{12}^{\mathrm{R}}(u + \omega) v_{21}^{\mathrm{I}}(u)]$$

$$F_2 = -[\Pi_2^{\mathrm{I}}(u + \omega) - \Pi_2^{\mathrm{I}}(u)][\Pi_1^{\mathrm{I}}(u + \omega) - \Pi_1^{\mathrm{I}}(u)][v_{12}^{\mathrm{R}}(u + \omega) v_{21}^{\mathrm{R}}(u) - v_{12}^{\mathrm{I}}(u + \omega) v_{21}^{\mathrm{I}}(u)] + [\Pi_2^{\mathrm{I}}(u + \omega) - \Pi_2^{\mathrm{I}}(u)][\Pi_1^{\mathrm{R}}(u + \omega) - \Pi_1^{\mathrm{R}}(u)] \times [v_{12}^{\mathrm{I}}(u + \omega) v_{21}^{\mathrm{R}}(u) + v_{12}^{\mathrm{R}}(u + \omega) v_{21}^{\mathrm{I}}(u)]$$

$$F_3 = 2[\Pi_2^{\mathrm{R}}(u + \omega) - \Pi_2^{\mathrm{R}}(u)]\{v_{12}^{\mathrm{R}}(u + \omega) v_{21}^{\mathrm{I}}(u) \Pi_1^{\mathrm{I}}(u + \omega) - v_{12}^{\mathrm{I}}(u + \omega) v_{21}^{\mathrm{R}}(u) \Pi_1^{\mathrm{I}}(u) - [\Pi_1^{\mathrm{R}}(u + \omega) - \Pi_1^{\mathrm{R}}(u)] v_{12}^{\mathrm{I}}(u + \omega) v_{21}^{\mathrm{I}}(u)\}$$

where the superscripts R and I represent the real and imaginary parts of the relative quantities. This result is rather complicated, but in principle can be evaluated analytically or numerically for specific problems and is valid for any temperature.

3. Temperature dependence of τ_{21}^{-1} in the DC limit

If the temperature is higher compared to the photon energy, we may use the zero-frequency result of (26) to study the temperature dependence of the interlayer relaxation process. First we notice that F_2, F_3 and the second term of F_1 all approach zero when $\omega \to 0$, and we obtain

$$\tau_{21}^{-1}(T) = \frac{\pi e^4}{2(m_1 n_2 + m_2 n_1)} \int \frac{q^2 \mathrm{d}^2 q}{4\pi} \int \mathrm{d}u \frac{\partial}{\partial u} \coth(\tfrac{1}{2}\beta u) |v_{21}(u)|^2 \Pi_1^{\mathrm{I}}(u) \Pi_2^{\mathrm{I}}(u). \tag{28}$$

Here (27) is only a special limiting case of a general frequency-dependent relaxation. By making use of (16), the interlayer coupling strength $|v_{12}(u)|^2$ can be rewritten as

$$|v_{12}(u)|^2 = \frac{|V_{12}|^2}{|\epsilon(q,u)|^2} + 2\frac{V_{12}}{|\epsilon(q,u)|^2} \mathrm{Re}\left(\frac{\tilde{\psi}_{21}}{\epsilon(q,u)}\right) + \frac{|\tilde{\psi}_{21}|^2}{|\epsilon(q,u)^2|^2}. \tag{29}$$

We shall discuss the contribution to $\tau_{12}^{-1}(T)$ due to each term in (27) below.

When the electron–phonon coupling is neglected, only the first term in (29) contributes. Furthermore, if we assume energy transfer during the collision is smaller compared to the temperature, static screening is applicable. By making use of the fact that, at low energy, $\Pi^{\mathrm{I}}(u)$ can be written as $F(q)u$, we immediately obtain

$$\tau_{21}^{-1}(T)|_{\mathrm{el-el}} = \frac{\pi e^4}{2(m_1 n_2 + m_2 n_1)} \int \frac{q^2 \mathrm{d}^2 q}{4\pi} F_1(q) F_2(q) \left|\frac{V_{21}}{\epsilon(0)}\right|^2 \int \mathrm{d}u u^2 \frac{\partial}{\partial u} \coth(\tfrac{1}{2}\beta u) \sim T^2. \tag{30}$$

Here $F_i(q) = m_i^2 \theta(k_{Fi}^2 - (q/2)^2)/q\sqrt{k_{Fi}^2 - (q/2)^2}$, where $\theta(x)$ is a Heaviside unit step function. It should be pointed out that the low-frequency Coulomb collision is always proportional to T^2 for virtually all systems [13, 16]; it is essentially due to the phase space allowed during the collision. Two-layer systems simply provide an excellent sample which demonstrates such T^2 behaviour. We also notice that $F_i(q)$ goes to zero as n_i goes to zero, therefore the interlayer collision frequency vanishes as the electron density on either layer approaches zero.

The contribution due to electron–phonon coupling is determined by both the second and third term in (29). Such effects were recently studied by Tso and co-workers [11]. In their work they assumed that the contribution due to the third term (termed as real phonon exchange) is negligible. Therefore they only consider the term proportional to $\tilde{\psi}$ (called the virtual phonon process). However, their result that the interlayer relaxation due to virtual phonon exchange is almost independent of layer separation a seems in contradiction with the second term in (29). It is clear that to leading order (under the assumption $q_s a \gg 1$, where q_s is the screening length), the contribution due to this term should have the same a dependence as that due to the first term. Our result of interlayer relaxation due to virtual phonon exchange can be written as

$$\tau_{21}^{-1}(T)|_{\mathrm{el-ph}} = \frac{\pi e^4}{2(m_1 n_2 + m_2 n_1)} \int \frac{q^2 \mathrm{d}^2 q}{4\pi} F_1(q) F_2(q) \frac{V_{21}^2}{\epsilon(0)^3} \times \int \mathrm{d}u u^2 \frac{\partial}{\partial u} \coth(\tfrac{1}{2}\beta u) \frac{\omega_q^2 (u^2 - \omega_q^2(1+\Delta))}{(u^2 - \omega_q^2(1+\Delta))^2 + (u\delta + \omega_q^2 \Gamma)^2}. \tag{31}$$

The numerical results of τ_{12}^{-1} for two different layer separations are presented in figure 2.

4. Frequency dependence of τ_{12}^{-1} at $T = 0$

We now consider the frequency dependence of $\tau_{21}^{-1}(\omega)$. If the temperature is low compared to the photon energy, we can perform our calculation by using the zero-temperature approximation, and obtain

$$\tau_{21}^{-1}(\omega) = \frac{\pi e^4}{2\omega(m_1 n_2 + m_2 n_1)} \int \frac{q^2 \mathrm{d}^2 q}{4\pi} \int_0^{\omega} \mathrm{d}u F(u, \omega - u). \tag{32}$$

At finite frequencies, it is not convenient to separate the contribution solely due to the electron–electron interaction because of the complex structure of $F(u, \omega - u)$. In this case, we shall calculate $\tau_{12}^{-1}(\omega)$ due to the total effective potential given by (15) or (16). The quantity $F(u, \omega - u)$ can be cast as sum of two terms, $F(u, \omega - u) = F_s(u, \omega - u) + F_c(u, \omega - u)$, where F_s is the contribution due to the single-particle excitation and F_c is the contribution due to the collective excitation of the coupled electron–phonon system. For the single-particle contribution, we write $F_s(u, \omega - u) = \tilde{F}_s(u, \omega - u)/|\tilde{\epsilon}(u)|^2|\tilde{\epsilon}(\omega - u)|^2$, where

$$\begin{aligned}
\tilde{F}(u, \omega - u) = {} & [\Pi_2^{\mathrm{I}}(\omega - u) + \Pi_2^{\mathrm{I}}(u)]\{[\Pi_1^{\mathrm{I}}(\omega - u) + \Pi_1^{\mathrm{I}}(u)](A_s + B_s) \\
& - [\Pi_1^{\mathrm{R}}(\omega - u) - \Pi_1^{\mathrm{R}}(u)](C_s - D_s)\} \\
& - [\Pi_2^{\mathrm{I}}(\omega - u) - \Pi_2^{\mathrm{I}}(u)]\{[\Pi_1^{\mathrm{I}}(\omega - u) - \Pi_1^{\mathrm{I}}(u)](A_s - B_s) \\
& - [\Pi_1^{\mathrm{R}}(\omega - u) - \Pi_1^{\mathrm{R}}(u)](C_s + D_s)\} \\
& + 2[\Pi_2^{\mathrm{R}}(\omega - u) - \Pi_2^{\mathrm{R}}(u)]\{D_s \Pi_1^{\mathrm{I}}(\omega - u) - \Pi_1^{\mathrm{I}}(u) C_s \\
& - [\Pi_1^{\mathrm{R}}(\omega - u) - \Pi_1^{\mathrm{R}}(u)]B_s\}
\end{aligned} \tag{33}$$

and

$$\tilde{\epsilon} = [1 - (V_{11} + \psi_{11})\Pi_1][1 - (V_{22} + \psi_{22})\Pi_2] - (V_{12} + \psi_{12})(V_{21} + \psi_{21})\Pi_1 \Pi_2. \tag{34}$$

The quantities A_s, B_s, C_s, D_s are given by

$$\begin{aligned}
A_s = {} & [(V_{21} + \psi_{21}^{\mathrm{R}}(\omega - u))\tilde{\epsilon}^{\mathrm{R}}(\omega - u) + \psi_{21}^{\mathrm{I}}(\omega - u)\tilde{\epsilon}^{\mathrm{I}}(\omega - u)] \\
& \times [(V_{21} + \psi_{21}^{\mathrm{R}}(u))\tilde{\epsilon}^{\mathrm{R}}(u) + \psi_{21}^{\mathrm{I}}(u)\tilde{\epsilon}^{\mathrm{I}}(u)]
\end{aligned}$$

$$\begin{aligned}
B_s = {} & [\psi_{21}^{\mathrm{I}}(\omega - u)\tilde{\epsilon}^{\mathrm{R}}(\omega - u) - (V_{21} + \psi_{21}^{\mathrm{R}}(\omega - u))\tilde{\epsilon}^{\mathrm{I}}(\omega - u)] \\
& \times [\psi_{21}^{\mathrm{I}}(u)\tilde{\epsilon}^{\mathrm{R}}(u) - (V_{21} + \psi_{21}^{\mathrm{R}}(u))\tilde{\epsilon}^{\mathrm{I}}(u)]
\end{aligned}$$

$$\begin{aligned}
C_s = {} & [\psi_{21}^{\mathrm{I}}(\omega - u)\tilde{\epsilon}^{\mathrm{R}}(\omega - u) - (V_{21} + \psi_{21}^{\mathrm{R}})\tilde{\epsilon}^{\mathrm{I}}(\omega - u)] \\
& \times [(V_{21} + \psi_{21}^{\mathrm{R}}(u))\tilde{\epsilon}^{\mathrm{R}}(u) + \psi_{21}^{\mathrm{I}}(u)\tilde{\epsilon}^{\mathrm{I}}(u)]
\end{aligned}$$

$$\begin{aligned}
D_s = {} & [(V_{21} + \psi_{21}^{\mathrm{R}}(\omega - u))\tilde{\epsilon}^{\mathrm{R}}(\omega - u) + \psi_{21}^{\mathrm{I}}(\omega - u)\tilde{\epsilon}^{\mathrm{I}}(\omega - u)] \\
& \times [\psi_{21}^{\mathrm{I}}(u)\tilde{\epsilon}^{\mathrm{R}}(u) - (V_{21} + \psi_{21}^{\mathrm{R}}(u))\tilde{\epsilon}^{\mathrm{I}}(u)].
\end{aligned}$$

The contribution due to the collective excitations, F_c, is given by

$$\begin{aligned}
F_c(u, \omega - u) = {} & [\Pi_2^{\mathrm{I}}(\omega - u) + \Pi_2^{\mathrm{I}}(u)]\{[\Pi_1^{\mathrm{I}}(\omega - u) + \Pi_1^{\mathrm{I}}(u)]B_c \\
& - [\Pi_1^{\mathrm{R}}(\omega - u) - \Pi_1^{\mathrm{R}}(u)](C_c - D_c)\} \\
& - [\Pi_2^{\mathrm{I}}(\omega - u) - \Pi_2^{\mathrm{I}}(u)]\{-[\Pi_1^{\mathrm{I}}(\omega - u) - \Pi_1^{\mathrm{I}}(u)]B_c \\
& - [\Pi_1^{\mathrm{R}}(\omega - u) - \Pi_1^{\mathrm{R}}(u)](C_c + D_c)\} \\
& + 2[\Pi_2^{\mathrm{R}}(\omega - u) - \Pi_2^{\mathrm{R}}(u)]\{D_c \Pi_1^{\mathrm{I}}(\omega - u) - \Pi_1^{\mathrm{I}}(u) C_c \\
& - [\Pi_1^{\mathrm{R}}(\omega - u) - \Pi_1^{\mathrm{R}}(u)]B_c\}
\end{aligned} \tag{35}$$

where

$$B_c = \delta\{\tilde{\epsilon}^R(\omega-u)\}(V_{21}+\psi_{21}^R(\omega-u))\left[V_{21}\tilde{\epsilon}^R(u)+(V_{21}+\psi_{21}^R(u))\tilde{\epsilon}^I(u)\right]|\tilde{\epsilon}(u)|^{-2}$$
$$+\delta\{\tilde{\epsilon}^R(u)\}(V_{21}+\psi_{21}^R(u))$$
$$\times\left[V_{21}\tilde{\epsilon}^R(\omega-u)+(V_{21}+\psi_{21}^R(\omega-u))\tilde{\epsilon}^I(\omega-u)\right]|\tilde{\epsilon}(\omega-u)|^{-2}$$

$$C_c = [(V_{21}+\psi_{21}^R(\omega-u))\tilde{\epsilon}^R(\omega-u)+\psi_{21}^I\tilde{\epsilon}^I(\omega-u)]$$
$$\times[V_{21}+\psi_{21}^R(\omega-u)]|\tilde{\epsilon}(\omega-u)|^{-2}\delta\{\tilde{\epsilon}^R(u)\}$$

$$D_c = [(V_{21}+\psi_{21}^R(u))\tilde{\epsilon}^R(u)+\psi_{21}^I\tilde{\epsilon}^I(u)][V_{21}+\psi_{21}^R(u)]|\tilde{\epsilon}(u)|^{-2}\delta\{\tilde{\epsilon}^R(\omega-u)\}.$$

In figure 3 we present the numerical of $\tau_{21}^{-1}(\omega)$ for two typical values of parameters.

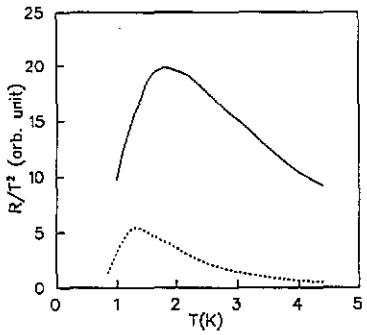

Figure 2. Plots of interlayer momentum relaxation rate $R = \tau_{21}^{-1}$ due to electron–phonon coupling as a function of temperature in the DC limit. Here $r_s = 1.0$ for both layers and the free-phonon linewidth is chosen as $0.0001E_F$. The full curve is for $k_Fa = 1.0$ and the dotted curve is for $k_Fa = 2.0$; the ratio v_s/v_F (where v_s is the sound velocity) is chosen to be 0.01.

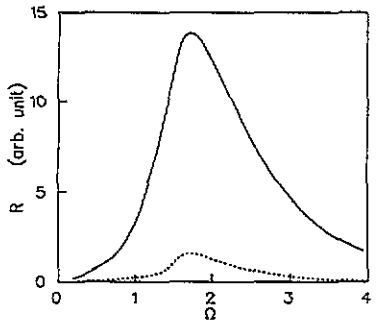

Figure 3. Plots of interlayer momentum relaxation rate R due to electron–phonon coupling as a function of normalized frequency, $\Omega = \omega/E_F$, at zero temperature. The full curve is for $k_Fa = 0.5$ and the dotted curve is for $k_Fa = 1.0$; other parameters are the same as in figure 2.

5. Results and discussions

In this work, our main concern is the dynamical conductivity of an electronic system consisting of two isolated parallel layers. We have calculated the off-diagonal component of the dynamical conductivity, which is directly related to the interlayer momentum relaxation rate. We obtain the rate $R \sim \tau_{21}^{-1}$ as a function of temperature and the frequency of photon field. Such momentum relaxation is due to both the electron–electron Coulomb interaction and the electron–phonon coupling. In the zero-frequency limit, the rate due to direct Coulomb interaction can be separated and the correction due to electron–phonon coupling is calculated explicitly. The temperature dependence of $R|_{el-ph}$ is similar to that obtained by the others. At low temperature, the rate goes to zero much faster than T^2 due to the vanishing phonon population. At high T, the rate depends very weakly on T. However, the rate depends on the layer separation very strongly, contrary to the findings of [11].

We have also studied the frequency dependence of $R(\omega)$ at zero temperature due to total effective potential. At low frequencies, $R(\omega)$ is again dominated by the direct Coulomb interaction and therefore exhibits a nearly ω^2 dependence. (It should be noted that an ω^2 or T^2 dependence is not limited to the present DQW structure, but is rather quite general, essentially because the density of states vanishes with ω). The deviation from ω^2, which is quite noticeable, is believed to be the effect of phonons. The collective excitation of the coupled electron–phonon system contributes most at around and above E_F. Here again, $R(\omega)$ depends strongly on the layer separation; when the separation is reduced by half, the rate increases by an order of magnitude.

We would like to point out that the previous conclusion [11] that the interlayer electron momentum relaxation is nearly independent of the layer separation seems incorrect to us. The electron–phonon coupling parameter, $\psi(q,\omega)$ (or $|M(q,\omega)|^2$ in [11]) is given by (5), (6) and (13), which is in general a strong separation dependent function. For 2D or quasi-2D electron systems, the processes only involve those phonons with negligible wavevector in the z direction and the coupling parameter is given by (14). Even if one adopts the jellium model, as in [11], a similar separation dependence should be obtained. In [11], the assumption that $|M(q,\omega)|^2 = 2\pi e^2\omega_\lambda/q^2$ is a probably oversimplified model which leads to the final result being separation independent. Intuitively, the coupling between two electrons should vanish when they are far apart, whether it is a direct coupling or through some other exchange mechanism.

In conclusion, the dynamical conductivity of a DQW in the 2D limit has been investigated. We hope that the frequency dependence presented here can also be studied experimentally. This will certainly provide another avenue to check the role of phonons in the electron transport in DQW structures.

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